OPTIMAL DESIGN OF STRUCTURES

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Abstract
This paper deals with an optimal design of slabs, regarding to maximum deflection constraints, using the ESO optimization method, established on an elimination of material due to the change of deflection or to the level of stress, alternatively. A weighted coefficient of sensitivity, which indicates the change of energy of structure as a result of decrease of depth of j-th element, has been used during the optimization procedure. Our aim is to reach as much as possible mainly uniformed stress state in slab. Some results of using these procedures are presented as well.

1 Introduction
Thanks the huge development computing techniques and therewith contiguous wide exploitation streamlined numerical method many new optimization methods, which passed especially in last two decades considerable development, are widely used in design construction process. These methods are mainly based on the Finite Element Method (FEM) basis, widely used by scientists and engineers, with an implementation of optimization procedures. This paper deals with some engineering techniques used in optimization process, well-known as structural optimization. That gives a relatively simple and easy answer on the question, how designed components obtain their shape and dimensions.

Structural optimization may be seen as a multidisciplinary science at the interface of engineering, mathematics, research and technology having an aim to reach the best design of a structure. A designer must take into account all aspects, positive and negative. Evolutionary Structural Optimization (ESO) was presented by their authors Z. M. Xie a G. P. Steven [1]. This relatively simple engineering method offers an approach used in structural optimization, in which the designed structure or its parts get step by step required shape or dimensions. It is based on a simple principle, in which the final optimal shape or dimensions are reached by successive remove or translation of ineffective material in designed structure.

2 Reduction of element thickness due to the sensitivity coefficient
Optimal design of structure is obviously done in respect to some predefined parameters, which must be fulfilled in designed structure. In this part we use the

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deflection constraint in several points of structure, which is used when it is necessary to limit deflections in many points.

The constraint in each of points may be written as follows

\[ |u_j| \leq |u_j^*| \quad (j = 1, m) \]  

(1)

where \( m \) - it the total number of prescribed deflections.

Using of a **weighted average sensitivity coefficient** one gets

\[ \alpha_j = \sum_{j=1}^{m} \lambda_j |\alpha_{ij}| \]  

(2)

where: \( \lambda_j = |u_j|/u_j^* \) - is a ratio of a current deflection to the description one.

Then

\[ \alpha_{ij} = |\{u_i\}^j \Delta[K_i]\{u_i\}| \]  

(3)

The sensitivity coefficient indicates the change of energy of structure as a result of decrease of depth of \( j \)-th element. In real, \( \alpha \) is a deformation energy of the element and may be determined very easily using the element stiffness matrix and the particular displacement vector. The design procedure consists of a set of steps. In each step, only a one percent of an original volume of structure may be moved or reduced. The selection of elements (their number depends on the 1 % of volume), which will undergo a reduction of thickness, is given by minimal weighted average sensitivity coefficients. Iteration cycle is stopped when the condition (1) is reached.

When only one constraint of deflection in a point is prescribed, the procedure works due to sensitivity coefficient (3), in case of multiply deflection constraint it is necessary to use the weighted average sensitivity coefficients (2), which consist of coefficients \( \lambda \) and \( \alpha_{ij} \).

### 3 Reduction of element thickness due to the stress state of \( \sigma_x, \sigma_y, \tau_{xy} \)

A reliable sign of potential structure failure is excessive stress or strain, inversely a reliable sign of inefficient material use is low stress or strain. Each part of an optimal structure has approximately the same (maximum) level of stress so the concept of described method consist of an introduction of the stress level based reducing ratio which assign what part of inefficient material may be moved or reject from the domain. Because of the sequential reducing of the thickness in low stressed parts of the slab and redistribution of inner forces the stress in whole structure becomes more and more uniform. In this paper some results of such technique based not on remove of the inefficient material but on reduce of its thickness in elements having the low level of stress are briefly presented. The stress level at each point can be measured by some sort of all the stress components. For our purpose the von Mises stress condition has been one of the most frequently used criteria (in case of isotropic material).The level of the maximum stress in slab elements (on upper or lower top) is defined by Von Mises stress \( \sigma_{VM} \) as:

\[ \sigma_{VM} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3 \tau_{xy}^2} \]  

(4)
where \( \sigma_x, \sigma_y \) – normal stress in directions \( x, y \),
\( \tau_{xy} \) – shear stress.

The level of stress intensity in each element may be given by the ratio of von Misses stress \( \sigma_{e}^{VM} \) in current element to maximal von Misses stress \( \sigma_{max}^{VM} \). At the end of each step of iteration all elements, which fulfil the following condition, may be reduced or moved out of the structure:

\[
\frac{\sigma_{e}^{VM}}{\sigma_{max}^{VM}} \leq R_i,
\]

where \( R_i \) is a current rejection coefficient.

An analysis of structure using the FEM is repeated in each cycle with the same rejection coefficient until no new elements may be moved. In this state an evolution coefficient (ER), which is added to the rejection coefficient as follows, is introduced:

\[
R_{i+1} = R_i + ER, \quad i = 0, 1, 2, 3, ..
\]

Having a new value of the rejection coefficient the next round of analysis of structure in iteration steps, in which more elements are moved until a new stationary state of stress is reached. This process is repeated until the prescribed optimum is reached.

This evolutionary procedure requires two parameters to be prescribed:

- the initial reducing ratio \( R_o \) (typical values of \( R_o = 1\% \));
- evolutionary ratio \( ER \), (typical value of \( ER = 1\% \)).

During the optimization procedure several constraints may be used. In our case, reducing the element thickness two different constraints may be used, either the constraint of limit stress (7) or the maximum deflection constraint (1).

\[
| \sigma_j | \leq \sigma_j^*
\]

Because of using the sensitivity coefficient we used the maximum deflection constraint (1).

### 4 Algorithm of optimization procedure:

The evolutionary procedure for such technique used in weight design of the slab with variable thickness may be described very briefly as follows:

- Static analysis of structure - meshing the structure using a fine mesh of finite elements,
- Solve the static equilibrium equation for the given load \{\textbf{P}\} and virtual unit loads corresponding to all the displacement constraints,
- Optimization analysis – may be done using two different procedures:
a) in respect to the sensitivity coefficient for each element using equation (3) or weighted average sensitivity coefficient (2) in respect to several constraints (1) the thickness in elements having the lowest sensitivity coefficient is reduced;

b) In respect to the rejection ratio (5) the ineffective part of material is moved.

- Repeat the previous steps until the displacement constraint conditions (4), (5) can no longer be satisfied.

The number of elements subjected to thickness reduction is determined by the step size $\Delta h$ of thickness reduction as well as the material reduction ratio $(MRR)$, which defines the maximum amount (volume or weight) of material what can be removed at each iteration step over the total initial material. A typical value for $MRR$ is 1%.

## 5 Optimal design of clamped slab

As an example we will optimize a clamped slab of 4*2 m. The free edge of length of 4 m is loaded in three points A, B, C by forces $P = 20kN$. Multilateral constraints of deflection are prescribed in load points Material properties are as follows: $E = 30GPa$, $\nu = 0.2$. The thickness of slab may vary in 0.2 – 0.1 m with a constant interval $\Delta t$.

Because of the symmetry of structure only a half of structure was modelled.

The aim was the design of such structure having the variable thickness by the minimum volume, which fulfils prescribed deflections on the free edge. Modelling the structure we used 800 triangular plate elements. The initial element thickness was $h_0 = 0.2$ m, initial maximum deflection 2.22mm. Limit of deflection in load points was rated 3.0 mm.

### 5.1 Design with two different thickness (optimization procedure by paragraph 2)

The optimization process described in par.2 was applied on structure given in Fig.1. The final topology of deflection is drawn in Fig. 2.a. The iteration process consisted of 21 iteration cycles. The maximum deflection in final structure of 79 % of initial volume was 2,965 mm.

![Fig.1: Topology of the model](image-url)
5.2 Design with two different thickness (optimization procedure by paragraph 3)

In Fig. 2.b some results of ESO optimization procedure due to par.3 using the von Misses condition (4) and (5) are presented. The final volume is 77% of initial volume of structure. Optimization procedure consist of 23 iterations, when maximum deflection 2,95 mm was reached.

In Fig. 4 - Fig. 6 some results of optimization procedures using five, six or eleven different thickness are presented.
Fig.4: Topology of five thickness

\[ \Delta t = 0.025 \]

\[ w_{A_{\text{max}}} = 2.924 \text{ mm} \]
reduct. 17 % of volume

\[ \sigma_{\text{max}}(763) = 6150.25 \text{ kPa} \]

\[ \Delta t = 0.02 \text{ m} \]

\[ w_{A_{\text{max}}} = 2.924 \text{ mm} \]
reduct. 17 % of volume

\[ \sigma_{\text{max}}(763) = 6150.25 \text{ kPa} \]

Obr.5: Topology of six thickness

\[ w_{A_{\text{max}}} = 2.975 \text{ mm} \]
reduct. 23 % of volume

\[ \sigma_{\text{max}}(763) = 6200.78 \text{ kPa} \]
In Fig. 7 some results of optimization procedures due to par. 2 and par. 3 applied to optimal design of a slab of 5*5m supported in four points loaded by a uniform load of q=10kN/m². Material properties: $E = 29\text{GPa}$, $\nu = 0.15$. The thickness of slab varied of 0.6 – 0.5 m with a constant interval $\Delta t = 0.1m$. As a constraint the maximum deflection in middle point 0.4 m was chosen. One of the result was that the procedure with von Mises condition failed because of minimum of difference between the stress state. Initial maximum deflection in middle point was 0.308m.

6 Optimal design of slab supported in four edge points

In Fig. 7 some results of optimization procedures due to par. 2 and par. 3 applied to optimal design of a slab of 5*5m supported in four points loaded by a uniform load of q=10kN/m². Material properties: $E = 29\text{GPa}$, $\nu = 0.15$. The thickness of slab varied of 0.6 – 0.5 m with a constant interval $\Delta t = 0.1m$. As a constraint the maximum deflection in middle point 0.4 m was chosen. One of the result was that the procedure with von Mises condition failed because of minimum of difference between the stress state. Initial maximum deflection in middle point was 0.308m.

<table>
<thead>
<tr>
<th>Thickness</th>
<th>Max Deflection</th>
<th>Reduction</th>
<th>Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.9306 mm</td>
<td>16%</td>
<td>6105.24 kPa</td>
</tr>
<tr>
<td>2</td>
<td>2.946 mm</td>
<td>22%</td>
<td>6105.24 kPa</td>
</tr>
</tbody>
</table>

**Obr.6: Topology of eleven thickness**

**Fig.7: Topology of two thickness**

a) full slab          b) one quarter
7 Conclusion

The presented engineering methods are available for optimal design of structures loaded by static load. Depending on the step of change one may get the very skipped or more continuous change of thickness. In both methods it is clear that the initial thickness may remain in points of load as well as in supports.

Acknowledgement

This paper was supported by State Grant Agency of Slovak Republic VEGA 1/0322/03.

References