ANALYSIS OF SENSITIVITY COEFFICIENTS IN EVOLUTIONARY STRUCTURAL OPTIMIZATION

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Abstract

This paper deals with an effective engineering technique used in structural optimization design by reducing element thickness. The optimization problem is formulated in static problems regarding the weight minimizing of the plate by reducing the element thickness. To identify these elements, the sensitivity coefficient for each element thickness reduction is calculated. At the elements with the lowest sensitivity coefficients the thicknesses can be reduced. Finally some results of a rectangular plate using optimization analysis are presented.

1 Introduction

Optimal design usually deals with achieving the best outcome of a given objective while satisfying certain restriction. In structural optimization it immediately appears that designers are often confronted with large scale problems. Thus several structures to be optimized are often intricate, and in order to obtain a good approximation they must be discretized into a large number of elements. Moreover several constraint functions are implicit functions of the design variables. In such conditions the cost of any general mathematical optimization methods is so high that it becomes prohibitive, the more so as the cost grows with the number of design variables. Therefore many less expensive techniques have been worked out and commonly used for a long time. Such techniques are not “pure” optimization methods in mathematic approach, they are generally based on intuition or computational experience. Nevertheless, in spite of a certain deficiency in rigour they often lead to acceptable designs. In a few words, they are approximate methods as well as techniques.

In our previous papers dealing with such methods [1],[2],[3] we informed on a new optimization technique introduced by Z. M. Xie and G. P. Steven [4], named Evolutionary Structural Optimization (ESO). Since 1992 it became to the answer of an effective method for the shape or size of optimal structures. ESO presents a new method of Structural Optimization, which gets over plenty of obstacles joined with classical techniques. The ESO method is based on the simple concept that by slowly removing

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inefficient material from a structure, the residual shape evolves in the direction of making the structure better.

2 Multiple Displacement Constraint

According to the general theory introduced in [4] rather than minimize the weight of a plate structure by removing elements (thus creating holes) we may better minimize the weight by reducing the thicknesses of some of the less efficient plate elements. To identify where these elements are, we need to calculate the sensitivity number for element thickness reduction.

For the static analysis in the modern engineering design process FEA is the best method, which is sufficiently flexible for analyses assorted problems. The static behavior is represented by the following equilibrium equation

\[ [K] \{u\} = \{P\}, \]  

(1)

where \([K]\) notes the global stiffness matrix, \(\{u\}\) is the nodal displacement vector and \(\{P\}\) is the nodal load vector.

It is assumed that the thickness of the \(i\)-th element is reduced from the old thickness \(h\) to the next lower thickness \((h - \Delta h)\). The change in the stiffness matrix of the structure is

\[ [\Delta K] = [\Delta K^i] = [K^i(h - \Delta h) - K^i(h)] \]  

(2)

where \(K^i(h)\) and \(K^i(h - \Delta h)\) note the stiffness matrices of the \(i\)-th element for the old thickness a new one, respectively. In our previous papers we accepted a constraint connected with \(j\)-th displacement component \(u_j\), as given in equation (3),

\[ |u_j| \leq u_j^* \]  

(3)

where \(u_j^*\) is a prescribed limit for \(|u_j|\).

For some structures it may be required that displacements at several different locations or directions be within prescribed limits. Such multiple displacement constraints may be given in the form

\[ |u_j| \leq |u_j^*| \quad (j = 1, m) \]  

(4)

where \(m\) is the total number of displacements with constraints.

There are various ways of dealing with multiple displacements constraints in the literature such as using Lagrangian multipliers. A simpler approach is to use the **weighted average of the expected changes** in the displacements due to element removal. The weighted average may be given in the form
\[
\alpha_j = \sum_{j=1}^{m} \lambda_j |\alpha_{ij}|
\]  \hspace{1cm} (5)

where \( \lambda_j = \frac{|u_j|}{u_j^*} \) is the ratio of the current displacement to constraint displacement in \( j \)-th element. The sensitivity coefficient for each element which indicates the effect of the element thickness reduction on \( u_j \), can be calculated as:

\[
\alpha_{ij} = -\{u_j^i\}^T \Delta [K_i] \{u_j^i\}
\]  \hspace{1cm} (6)

The evolutionary procedure for optimization with multiple displacement constraints is given as follows:

- Discretize the structure using a fine mesh of finite elements,
- Solve the static equilibrium equation (1) for the given load \( \{P\}\) and virtual unit loads corresponding to all the displacement constraints,
- Calculate the sensitivity coefficient for each element using equation (6),
- Reduce the thicknesses of number of elements which have the lowest sensitivity coefficients.
- Repeat steps 2 to 4 until one of the conditions (3) can no longer be satisfied.

The number of elements subjected to thickness reduction is determined by the step size \( \Delta h \) of thickness reduction as well as the material removal ratio (MRR), which defines the maximum amount (volume or weight) of material what can be removed at each iteration step over the total initial material. A typical value for MRR is 1%. As far as we require fulfillment of multiply constraints (in our case displacements in \( j \)-nodes \( j = 1, m \)), then in 3th step of the current iteration it is necessary to calculate weighted sensitivity coefficient, which includes coefficient \( \lambda_j \) a sensitivity coefficient \( \alpha_{ij} \).

### 3 Example of Multiple Displacement Constraint Optimization

Fig. 1 shows a rectangular plate \( (4 \text{ m} \times 2 \text{ m}) \) which is clamped at one long edge. There are three point loads, each of 20 kN, acting normal to the plate at the two corners and in the middle. Multiple displacements constraints are imposed on the out-of-plane displacements at the loaded points. The Young’s modulus \( E = 30 \text{ GPa} \) and Poisson’s ratio \( \nu = 0,2 \) are assumed. The thickness of the plate \( h \) may vary from 0,1 to 0,2 m with equal intervals \( \Delta h \) in between. Because of symmetry, only half of the cantilever is modeled. Maximum of displacements \( w_{max} = 0,003 \text{ m} \) is prescribed.

Number of elements belonging to thickness reduction is defined by:

- **Interval of thickness reduction \( \Delta h \)**
- **Prescribed material removal ratio (MRR) – typical value is 1%**.
In case we specify $\Delta h = 0,1 \ m$ (two different thickness), the requirement (1) is fulfilled in 21st iteration step. The maximum displacement in point A is $w_{\text{max}} = 2,965 \ mm < 3,00 \ mm$. The final volume of the plate is 79 % of the initial volume. The final topology of elements is given in Fig. 2a.

In case we specify $\Delta h = 0,05 \ m$ (three different thickness), the requirement (1) is fulfilled in 19th iteration step. The maximum displacement in point A is $w_{\text{max}} = 2,9415 \ mm < 3,00 \ mm$. The final volume of the plate is 81 % of the initial volume. The final topology of elements is given in Fig. 2b.

In case we specify $\Delta h = 0,025 \ m$ (five different thickness), the requirement (1) is fulfilled in 18th iteration step. The maximum displacement in point A is $w_{\text{max}} = 2,975 \ mm < 3,00 \ mm$. The final volume of the plate is 82 % of the initial volume. The final topology of elements is given in Fig. 2c.

In case we specify $\Delta h = 0,02 \ m$ (six different thickness), the requirement (1) is fulfilled in 17th iteration step. The maximum displacement in point A is $w_{\text{max}} = 2,9244 \ mm < 3,00 \ mm$. The final volume of the plate is 83 % of the initial volume. The final topology of elements is given in Fig. 3a.

In case we specify $\Delta h = 0,01 \ m$ (eleven different thickness), the requirement (1) is fulfilled in 16th iteration step. The maximum displacement in point A is $w_{\text{max}} = 2,9306 \ mm < 3,00 \ mm$. The final volume of the plate is 84 % of the initial volume. The final topology of elements is given in Fig. 3b.

Fig. 2: Final topology of elements in case of 2,3 and 5 thickness

Fig. 3: Final topology of elements in case of 6 and 11 thickness
History of displacements changes during particular iteration steps for different values of $\Delta h = 0.01m; 0.02m; 0.025m; 0.05m$ and $0.1 m$ i.e. for 11, 6, 5, 3, and 2 thickness are displayed in Fig. 4. The initial displacement in point A is 2.22 mm and constrained displacement condition in both points A and B is 3 mm.

![Plot of displacement changes](image)

**Fig 1: History of displacement changes**

**4 Conclusion**

As it is clear from previous figures, the finer value $\Delta h$ evokes finer change of particular thickness of plate (in the last case is almost continuous). The choice of $\Delta h$ has a relevant influence to the number of iteration cycles, in which the final constraint condition will be achieved. As it is seen, in case of heavy mesh of $\Delta h = 0.1 m = 2$ thickness it is necessary to pass 21 iteration cycles to achieve the final constraint, in case of fine mesh of $\Delta h$ decreases the number of iteration cycles (when $\Delta h = 0.01 m = 11$ thickness, the procedure finish in 16th iteration cycle.

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**References**

