An Introduction to the Mechanics of Soils and Foundations

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An Introduction to
THE MECHANICS OF SOILS AND FOUNDATIONS
Through Critical State Soil Mechanics

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This book is about the behaviour of engineering soils and simple geotechnical structures such as foundations and slopes and it covers most of the theoretical geotechnical engineering content of a degree course in civil engineering. The book is aimed primarily at students taking first degree courses in civil engineering but it should also appeal to engineers, engineering geologists and postgraduate students wishing for a simple and straightforward introduction to the current theories of soil mechanics and geotechnical engineering. Although it deals specifically with soils and soil mechanics many of the theories and methods described apply also to rocks and rock mechanics.

The teaching and practice of geotechnical engineering has undergone significant changes in the past 25 years or so, both in the development of new theories and practices and in the standing of the subject within the civil engineering curriculum. Geotechnical engineering is now regarded as one of the major disciplines in civil engineering analysis (the others being hydraulics and structures). The most important development, however, has been the unification of shearing and volumetric effects in soil mechanics in the theories known generally as critical state soil mechanics and application of these theories in geotechnical analysis. In this book, unlike most of the other contemporary books on soil mechanics, the subject is developed using the unified theories right from the start, and theories for stability of foundations and slopes are developed through the upper and lower bound plasticity methods as well as the more commonly used limit equilibrium method. This is an up-to-date approach to soil mechanics and geotechnical engineering and it provides a simple and logical framework for teaching the basic principles of the subject.

The term ‘critical state soil mechanics’ means different things to different people. Some take critical state soil mechanics to include the complete mathematical model known as Cam Clay and they would say that this is too advanced for an undergraduate course. My view is much simpler, and by critical state soil mechanics I mean the combination of shear stress, normal stress and volume into a single unifying framework. In this way a much clearer idea emerges of the behaviour of normally consolidated and overconsolidated soils during drained and undrained loading up to, and including, the ultimate or critical states. It is the relationship between the initial states and the critical states that largely determines soil behaviour. This simple framework is extremely useful for teaching and learning about soil mechanics and it leads to a number of simple analyses for stability of slopes, walls and foundations.

This book is based on courses of lectures given to undergraduate students in civil engineering at City University. In the first year students take a course in geology and they also take a course in mechanics of materials within which there are six to eight lectures on soil mechanics and geotechnical engineering. These lectures cover the whole of the conventional syllabus (classification, seepage, strength, consolidation, bearing capacity and settlement, slope stability and earth pressure) but at lightning speed. The object is to introduce the students to the concepts and vocabulary of geotechnical engineering within the context of conventional mechanics of materials and structures and with reference to their everyday, childhood experiences of playing with
sand, flour, plasticine and other soil-like materials so that, as the course develops in later years, they can relate particular topics into the whole scheme of civil engineering.

In the second year the students take a major course of lectures (with several laboratory sessions) in theoretical soil mechanics and geotechnical engineering. This is based on my earlier books—The Mechanics of Soils (with Peter Bransby) and Foundations and Slopes. This course depends entirely on the unification of shearing and volumetric effects which is introduced right from the start (and had been in the first year), although the phrase ‘critical state soil mechanics’ is rarely used. Theoretical soil mechanics is taken up to the development of a complete state boundary surface but stops short of the mathematical treatment ofCam clay. Stability problems are solved using upper and lower bound methods and these are then used to introduce limit equilibrium methods and standard tables and charts for bearing capacity, slope stability and earth pressure. In the third year the course covers practical aspects of geotechnical engineering through a series of lectures and projects on topics such as ground investigation, foundations, slopes, retaining walls and embankment designs.

This book covers the material in the second-year course (and also that summarized in the first year). It does not deal specifically with the practical aspects of geotechnical engineering which are introduced in the third year and are, in any case, generally better learned through working in practice with experienced engineers. This book should provide the basic text for an undergraduate course, but students will have to consult other books and publications to find more detailed coverage of particular topics such as laboratory testing, seepage, slope stability and foundation design.

The treatment of soil mechanics and geotechnical engineering in this book is simple, straightforward and largely idealized. I have tried to relate the behaviour of soils and geotechnical structures to everyday experiences, encouraging students to perform simple experiments themselves at home, on holiday and in a basic soil mechanics laboratory. I have described some simple tests which are designed to demonstrate the basic principles rather than generate highly accurate results. Only a few details are given of the apparatus and procedures since engineers should be trained to design and build simple equipment and work out how to make observations and analyse results themselves.

To illustrate the basic nature of soil strength and stiffness I have described the behaviour of soils in oedometer tests and in ideal shear tests in order to separate the effects of normal stress and compression from the effects of shearing and distortion. I have also described the behaviour of soils in triaxial tests, as these are the best tests to evaluate soil parameters. Readers will notice that I have not included data from tests on real soils or case histories of construction performance. This is quite deliberate and is common practice in undergraduate texts on structures, hydraulics, concrete and so on. As the book is intended primarily as an undergraduate teaching text it is kept simple and straightforward. The basic soil mechanics theories have been clearly demonstrated in earlier books from Critical State Soil Mechanics by Schofield and Wroth in 1968 to Soil Behaviour and Critical State Soil Mechanics by Muir Wood in 1991, and almost everything in this book follows from these well-established theories.

Throughout I have dealt with simple theories and idealizations for soil behaviour. I am very well aware that many natural soils behave in ways that differ from these idealizations and that there are a number of additional factors that may influence the design and analysis of geotechnical structures. Nevertheless, I am convinced that for the purposes of teaching the fundamental principles to students it is better to maintain the simplicity of the idealized treatment, provided always that they appreciate that it is idealized. At many points in the text I have indicated where the behaviour of various natural soils may depart significantly from the idealized behaviour. I expect that individual lecturers will bring in other examples of the behaviour of natural soils drawn from their own experiences, but I hope that they would discuss these within the simple framework described in this book.
At the end of most chapters there is a short summary of the main points covered in the chapter and, in most cases, simple worked examples and exercises that illustrate the theories developed in the text. There is also a short selection of books and articles for further reading and a list of specific references quoted in the text.

The courses at City University which form the basis of this book were developed jointly with my colleagues Neil Taylor, Matthew Coop and John Evans and I am grateful to them for their contributions and for their comments and criticisms. I am grateful also for the very detailed comments that I received from many friends and colleagues, including Mark Allman, Eddie Bromhead, Peter Fookes, Charles Hird, Marcus Matthews, Sarah Stallebrass and Giulia Viggiani. The typing was shared between Anne-Christine Delalande and Robert Atkinson.

John Atkinson
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The SI system of units has been used: the basic units of measurement are:

<table>
<thead>
<tr>
<th>Length</th>
<th>m</th>
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<tbody>
<tr>
<td>Time</td>
<td>s</td>
</tr>
<tr>
<td>Force</td>
<td>N</td>
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</table>

Some useful derived units are:

<table>
<thead>
<tr>
<th>Velocity</th>
<th>m/s</th>
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<tr>
<td>Acceleration</td>
<td>m/s²</td>
</tr>
<tr>
<td>Stress (pressure)</td>
<td>kN/m² = kiloPascal = kPa</td>
</tr>
<tr>
<td>Unit weight</td>
<td>kN/m³</td>
</tr>
</tbody>
</table>

Unit force (1 N) gives unit mass (1 kg) unit acceleration (1 m/s²). The acceleration due to the Earth’s gravity is \( g = 9.81 \text{ m/s}² \); hence the force due to a mass of 1 kg at rest on Earth is 9.81 N. (Note: there are about 10 apples in 1 kg: hence a stationary apple gives rise to a force of about 1 N acting vertically downwards.)
As in most branches of science and engineering, geotechnical engineering uses mathematics and symbols to develop general theories. Because the English alphabet has a limited number of characters, much of the Greek alphabet is used.
GLOSSARY OF SYMBOLS

Stress and strain parameters

One-dimensional compression and shear tests:

\( \tau' \) shear stress
\( \sigma' \) normal stress
\( \gamma \) shear strain
\( \varepsilon_v \) volumetric strain = normal strain

Triaxial tests:

\( q' = (\sigma'_a - \sigma'_i) \) deviatoric stress
\( p' = \frac{1}{3}(\sigma'_a + 2\sigma'_i) \) mean normal stress
\( \varepsilon_s = \frac{2}{3}(\varepsilon_a - \varepsilon_r) \) shear strain
\( \varepsilon_v = \varepsilon_a + 2\varepsilon_r \) volumetric strain

Superscripts for strains

\( e \) elastic
\( p \) plastic

Subscripts for states

0 initial state (i.e. \( q_{0}, p_{0}, v_{0} \))
f critical state (i.e. \( q_{f}, p_{f}, v_{f} \))
p peak state (i.e. \( q_{p}, p_{p}, v_{p} \))

Subscripts for axes

\( z, h \) vertical and horizontal
\( a, r \) axial and radial

Normalizing parameters

\( \ln p'_{c} = (\Gamma - v)/\lambda \)
\( v_{\lambda} = v + \lambda \ln p' \)
\( \log \sigma'_{c} = (e_{\Gamma} - e)/C_{c} \)
\( e_{\lambda} = e + C_{e} \log \sigma' \)

A area
A activity
B breadth or width
\( C_{c} \) slope of the normal compression line
\( C_{s} \) slope of a swelling and recompression line
D depth
\( D_{r} \) relative density
E Young's modulus (\( E' \) for effective stress; \( E_{u} \) for undrained loading)
\( F_{s} \) factor of safety
\( G_{s} \) specific gravity of soil grains
$G$  shear modulus ($G'$ for effective stress; $G_u$ for undrained loading)

$H$  height or thickness

$H$  maximum drainage path

$I_{p}$  influence coefficient for stress

$I_{p'}$  influence coefficient for settlement

$K$  bulk modulus

$K_o$  coefficient of earth pressure at rest

$K_a$  coefficient of active earth pressure

$K_p$  coefficient of passive earth pressure

$L$  length

$LL$  liquid limit

$LI$  liquidity index

$N$  normal force

$N_s, N_r, N_q$  bearing capacity factors

$P$  potential

$P$  force on retaining wall

$P_a$  force due to active pressure

$P_p$  force due to passive pressure

$P_n$  force due to free water

$Q$  flow (volume)

$Q_p$  pile load

$Q_b$  pile base resistance

$Q_s$  pile shaft resistance

$R$  radius

$S_c$  stress state parameter $= p'_{c}/p'$

$S_v$  volume state parameter $= v_c - v$

$T$  shear force

$T_c$  time factor for one-dimensional consolidation

$T_r$  time factor for radial consolidation

$U$  force due to pore pressures

$U_c$  average degree of consolidation

$V$  volume

$V_s$  volume of water

$V_s'$  volume of soil grains

$V$  velocity (of seepage)

$W$  work

$W$  weight

$W_w$  weight of water

$W_s$  weight of soil grains

$b$  thickness or width

$c'$  cohesion intercept in Mohr–Coulomb failure criterion

$c_r$  coefficient of consolidation

$e$  voids ratio

$e_0$  voids ratio of normally consolidated soil at $p' = 1.0$ kPa

$e_x$  voids ratio of overconsolidated soil at $p' = 1.0$ kPa

$e_r$  voids ratio of soil on the critical state line at $p' = 1.0$ kPa

$g$  shear modulus for states inside the state boundary surface

$h_w$  height of water in standpipe

$i$  slope angle
critical slope angle
hydraulic gradient
critical hydraulic gradient
coefficient of permeability
coefficient of compressibility for one-dimensional compression
rate of seepage
bearing pressure
bearing capacity
net bearing pressure
allowable bearing pressure
radius
pore pressure coefficient
length along a flowline
undrained strength
time
pore pressure
steady state pressure
excess pore pressure
specific volume
specific volume of overconsolidated soil at \( p' = 1.0 \text{ kPa} \)
water content
specific volume of soil on the critical state line at \( p' = 1.0 \text{ kPa} \)
large increment of
slope of CSL projected to \( q':p' \) plane
specific volume of normally consolidated soil at \( p' = 1.0 \text{ kPa} \)
sum of
adhesion factor for pile friction
unit weight
dry unit weight
unit weight of water (\( = 9.81 \text{ kN/m}^3 \))
small increment of
angle of friction between structure and soil
slope of swelling and recompression line
slope of normal consolidation line and CSL
Poisson's ratio (\( v \) for drained loading, \( v_u = \frac{1}{2} \) for undrained loading)
settlement
consolidation settlement
initial settlement
settlement at time \( t \)
final consolidation settlement
angle of friction
allowable friction angle
critical state friction angle
peak friction angle
angle of dilation
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SUBJECT INDEX
An Introduction to the Mechanics of Soils and Foundations covers the full undergraduate course in geotechnical engineering. It also provides a concise introduction to modern soil mechanics for advanced-course students and for practising engineers. The book sets out the basic theories of soil mechanics in a clear and simple way, combining both classical and critical state theories. Through up-to-date critical state soil mechanics, the reader is provided with a clear and concise theory for understanding the fundamental features of soil behaviour relating strength, stiffness, dilation and cohesion in a single framework. Theories for stability of slopes, foundations and retaining walls are developed through the simple upper- and lower-bound plasticity methods, as well as through the classical limit-equilibrium method. In addition, the book includes short introductions to basic mechanics and to geology, thus relating the material in geotechnical engineering to other topics in the civil engineering curriculum.

The text is designed for ease of use with short, focused chapters each dealing with a particular topic or aspect of soil behaviour. In this way, the author ensures a readable and accessible text while maintaining a continuous thread running though the book as theory develops into application. The treatment of soil mechanics is essentially theoretical, but it is not highly mathematical and soil behaviour is represented by relatively simple equations with clearly defined parameters.

Additional features
- Covers practical topics, such as geology, natural soils, site investigation and laboratory testing.
- Introduces advanced topics, such as Cam clay, non-linear soil stiffness and centrifuge modelling.
- Key points illustrated by worked examples and simple experimental demonstrations.

Professor John Atkinson holds a personal chair in Soil Mechanics at City University, London, where he established the Geotechnical Research Centre, now internationally renowned for research in soil mechanics and geotechnical engineering. He is recognized as a leading authority in the field and has acted as a consultant to industry on major projects in the UK and abroad. He lectures widely on soil mechanics, and he is the author of The Mechanics of Soils (with P.L. Bransby) and Foundations and Slopes, also published by McGraw-Hill.